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Formulas for accelerator physics and synchrotron radiation

Curvature radius of ultra relativistic particles in a storage ring:

$$\frac{1}{r} = e \cdot \frac{B}{p} = \frac{c \cdot e \cdot B}{c \cdot p} = c \cdot e \cdot \frac{B}{E}$$

$$\frac{1}{r[m]} = 0.299792458 \cdot \frac{B[T]}{E[GeV]}; \quad 1[T] = 1[Vsec/m^2]$$

$(Example: \ r = 12.1849m; \ B = 1.221884T; \ E = 4.5GeV)$
$(Example: \ r = 12.1849m; \ B = 1.450885T; \ E = 5.3GeV)$
$(Example: \ r = 191.7295m; \ B = 0.20877T; \ E = 12GeV)$
$(Example: \ r = 608m; \ B = 0.15142T; \ E = 27.6GeV)$

Quadrupole strength:

$$k = e \cdot \frac{g}{p} = \frac{c \cdot e \cdot g}{c \cdot p} = c \cdot e \cdot \frac{g}{E}$$

$$k[m^{-2}] = 0.299792458 \cdot \frac{g[T/m]}{E[GeV]}; \quad B = g \cdot x$$

$(Example: \ k = 0.6662m^{-2}; \ g = 10T/m; \ E = 4.5GeV)$

Sextupole strength:

$$m = e \cdot \frac{g'}{p} = \frac{c \cdot e \cdot g'}{c \cdot p} = c \cdot e \cdot \frac{g'}{E}$$

$$m[m^{-3}] = 0.299792458 \cdot \frac{g'[T/m^2]}{E[GeV]}; \quad B = \frac{1}{2} \cdot g' \cdot x^2$$

$(Example: \ m = 6.662m^{-3}; \ g' = 100T/m^2; \ E = 4.5GeV)$

Field index of a synchrotron magnet:

$$n = -\frac{r}{B_0} \cdot \frac{\Delta B_z}{\Delta x}$$

$(Example: \ n = -22.26; \ r = 7.65; \ B_0 = 1.003T; \ \Delta B_z/\Delta x = 2.919T/m)$

Focal length of a quadrupole:

$$\frac{1}{f} = \sqrt{k} \cdot \sin(\sqrt{k} \cdot l)$$

$$\approx k \cdot l \quad for\ small\ k\ and\ l$$

$(Example: \ f = 2.16m; \ k = 0.7m^{-2}; \ l = 0.7m)$

Wiggler parameter k:

$$k = \frac{e}{2\pi \cdot m_e \cdot c} \cdot B_{max} \cdot \lambda$$

$$k = 0.9337287 \cdot B_{max}[T] \cdot \lambda[cm]; \quad 1[kg] = 1[VAsec^3/m^2]$$

$(Example: \ k = 7.40; \ B_{max} = 0.6T; \ \lambda = 13.2cm)$
$(Example: \ k = 28.0; \ B_{max} = 1.25T; \ \lambda = 24cm)$
$(Example: \ k = 1.61; \ B_{max} = 0.55T; \ \lambda = 3.14cm)$

Focal length of a wiggler (vertical plane):

$$\frac{1}{f} = \frac{\tan\epsilon}{r} = \frac{2\pi^2 \cdot N \cdot k^2}{\lambda \cdot \gamma^2}; \quad \gamma = \frac{E}{E_0}$$

$$= \frac{1}{2} c^2 \cdot e^2 \cdot L \cdot \frac{B_{max}^2}{E^2}$$

$$\frac{1}{f[m]} = 0.04493776 \cdot L[m] \cdot \frac{B_{max}[T]^2}{E[GeV]^2}$$

$(Example: \ f = 593m; \ L = 2.112m; \ B_{max} = 0.6T; \ E = 4.5GeV)$
$(Example: \ f = 113m; \ L = 4.00m; \ B_{max} = 1.0T; \ E = 4.5GeV)$

Maximal electron deflection angle in a wiggler or undulator:

$$\Delta = \frac{k}{\gamma} = k \cdot \frac{E_0}{E} = \frac{c \cdot e}{2\pi} \cdot \lambda \cdot \frac{B_{max}}{E}$$

$$\Delta = 0.04771345 \cdot \lambda[m] \cdot \frac{B_{max}[T]}{E[GeV]} \quad for\ electrons$$

$(Example: \ \Delta = 3.18mrad; \ k = 28; \ E = 4.5GeV)$
$(Example: \ \Delta = 1.50mrad; \ k = 13.2; \ E = 4.5GeV)$
$(Example: \ \Delta = 3.18mrad; \ \lambda = 0.24m; \ B_{max} = 1.25T; \ E = 4.5GeV)$

Number of ultra relativistic particles in a storage ring:

$$n = \frac{U \cdot I}{c \cdot Q}; \quad I = \frac{n \cdot Q}{T}; \quad c = \frac{U}{T}$$

$(Example: \ n = 600 \cdot 10^9; \ I = 0.1A; \ U = 288m; \ Q = e)$
$(Example: \ n = 6.02 \cdot 10^9; \ I = 1mA; \ U = 289.2m; \ Q = e)$

Energy of a particle:

$$E = \sqrt{c^2 \cdot p^2 + E_0^2}$$

$$(Example: \ E = 3.7GeV; \ p = 3699.999965GeV/c; \ electrons)$$
$$(Example: \ E = 7.062603GeV; \ p = 7GeV/c; \ protons)$$

Kinetic energy of a particle beam:

$$E_{kin}[J] = 1.60217733 \cdot 10^{-19} \cdot E_{kin}[eV]$$

$$(Example: \ E_{total} = 2.89MJ; \ n = 2.2 \cdot 10^{13}; \ E = 820GeV)$$

$$E_{kin}[J] = E_{kin}[m^2 kg/sec^2]$$

$$(Example: \ E_{total} = 3.09MJ; \ n = 1; \ m = 2000kg; \ v = 200km/h)$$

α, β and γ function:

$$\gamma = \frac{1+\alpha^2}{\beta}; \quad \alpha = -\frac{\beta_r'}{2}$$

β function at the end of a straight section of the length L:

$$\beta_L = \beta_0 - 2L \cdot \alpha_0 + L^2 \cdot \frac{1+\alpha_0^2}{\beta_0}$$

Change of position and angle because of a kick in a storage ring:

$$dz = \frac{1}{2}\alpha_k \sqrt{\beta_k \cdot \beta_z} \cdot \frac{\cos(\pi Q_z - 2\pi|\varphi_z - \varphi_k|)}{\sin(\pi Q_z)}$$

$$dz' = -\frac{1}{2}\alpha_k \frac{\sqrt{\frac{\beta_k}{\beta_z}}}{\sin(\pi Q_z)} \cdot (\alpha_z \cdot \cos(\pi Q_z - 2\pi|\varphi_z - \varphi_k|) - \frac{\varphi_z - \varphi_k}{|\varphi_z - \varphi_k|} \cdot \sin(\pi Q_z - 2\pi|\varphi_z - \varphi_k|))$$

Change of position and angle because of a local bump kick:

$$dz = \alpha_k \sqrt{\beta_k \cdot \beta_z} \cdot \sin(\varphi_z - \varphi_k)$$

$$dz' = \alpha_k \sqrt{\frac{\beta_k}{\beta_z}} \cdot (\cos(\varphi_z - \varphi_k) - (-\frac{\beta_z'}{2}) \cdot \sin(\varphi_z - \varphi_k))$$

$$(Example: \ dz = 2mm; \ \alpha_k = 0.1mrad; \ \beta_k = \beta_z = 20m/rad; \ \varphi_z - \varphi_k = \pi/2)$$

Orbit lengthening because of a horizontal kick in a storage ring:

$$dl = D_x \cdot \alpha_k$$

Enlargement of the quadrupole strength in F(D) quadrupoles leads there to a reduction of the $\beta_x(\beta_z)$ function

Tune shift due to a change of the quadrupole strength:

$$\Delta Q = \frac{1}{4\pi} \cdot \beta \cdot L \cdot \Delta k$$

$(Example: \ \Delta Q = 0.1114; \ \beta = 20m; \ L = 0.7m; \ \Delta k = 0.1m^{-2})$

Vertical tune shift due to a wiggler:

$$\Delta Q_z = \frac{\pi \cdot \beta_z \cdot N \cdot k^2}{2 \cdot \gamma^2 \cdot \lambda} = \frac{c^2 \cdot e^2 \cdot \beta_z \cdot L \cdot B_{max}^2}{8\pi \cdot E^2}$$

$$\Delta Q_z = 0.003576033 \cdot \beta_z[m] \cdot L[m] \cdot \frac{B_{max}[T]^2}{E[GeV]^2}$$

$(Example: \ \Delta Q_z = 0.0017; \ \beta_z = 13m; \ L = 2.112m; \ B_{max} = 0.60T; \ E = 4.5GeV)$
$(Example: \ \Delta Q_z = 0.0106; \ \beta_z = 16m; \ L = 2.400m; \ B_{max} = 1.25T; \ E = 4.5GeV)$
$(Example: \ \Delta Q_z = 0.0170; \ \beta_z = 24m; \ L = 4.000m; \ B_{max} = 1.00T; \ E = 4.5GeV)$

Fraction of the tune of a storage ring:

$$frac(Q) = \frac{1}{2\pi} \arccos(\frac{1}{2} trace(M)) = \frac{1}{2\pi} \arccos(\frac{1}{2}(M11 + M22))$$

Tune of a storage ring:

$$Q = \frac{1}{2\pi} \cdot \int \frac{1}{\beta} ds$$

Chromaticity of a storage ring:

$$\frac{\Delta Q}{\frac{\Delta E}{E}} = \frac{1}{4\pi} \int (k - m \cdot D) \cdot \beta \cdot dl$$

$$(k < 0 : F - quadrupole, m < 0 : F - sextupole)$$

Momentum compaction factor of a storage ring:

$$\alpha_c = \frac{\frac{\Delta L}{L}}{\frac{\Delta p}{p}} = \frac{\frac{\Delta T}{T}}{\frac{\Delta E}{E}} \ (ultra\ relativistic)$$

$$\alpha_c = \frac{\frac{\Delta T}{T}}{\frac{\Delta p}{p}} + \frac{1}{\gamma^2} \ (relativistic)$$

Transition energy:

$$\gamma_{tran} = \frac{E}{E_0} = \frac{1}{\sqrt{\alpha_c}}$$

$(Example: \; E = 3.9MeV; \; electrons; \; \alpha_c = 0.017)$
$(Example: \; E = 8.77GeV; \; protons; \; \alpha_c = 0.01145)$

Emittance after adiabatic damping in a storage ring:

$$\epsilon_x \sim \frac{1}{\beta \cdot \gamma} \; (relativistic)$$

$$\epsilon_x \sim \frac{1}{\gamma} = \frac{E_0}{E} \; (ultra \; relativistic)$$

Emittance of a radiation damped beam in a storage ring:

$$\epsilon_x \sim E^2$$

Emittance dependance in a storage ring:

$$\Delta\epsilon_x \sim \frac{D_x^2}{\beta_x \cdot abs(r)}, \; if \; D'_x = 0, \; \alpha_x = 0 \; and \; D_x << abs(r)$$

Beam dimensions in an electron storage ring:

$$\sigma_x = \sqrt{\epsilon_x \cdot \beta_x + (D_x \cdot \sigma_e)^2}$$

$$\sigma_z = \sqrt{\epsilon_z \cdot \beta_z}$$

$$\sigma'_x = \sqrt{\epsilon_x \frac{1 + \alpha_x^2}{\beta_x} + (D'_x \cdot \sigma_e)^2}$$

$$\sigma'_z = \sqrt{\epsilon_z \frac{1 + \alpha_z^2}{\beta_z}}$$

$(Example: \; \sigma_x = 3.0mm; \; \epsilon_x = 0.4mm \cdot mrad; \; \beta_x = 20m/rad;$
$D_x = 1m; \; \sigma_e = 0.001)$
$(Example: \; \sigma'_z = 28\mu rad; \; \epsilon_z = 0.008mm \cdot mrad; \; \beta_z = 10m/rad; \; \alpha_z = 0)$

Touschek scattering means elastic electron electron scattering within a bunch

Beam lifetime due to inelastic electron scattering with residual gas nucleons (bremsstrahlung production):

$$\tau_B = \frac{1}{\alpha \cdot c \cdot \frac{(4r_e)^2}{3} \cdot Z(Z+1) \cdot \ln(183 \cdot Z^{-1/3}) \cdot (-\ln\epsilon - \frac{5}{8}) \cdot N_g}$$

$$\tau_B[sec] = \frac{1.079323 \cdot 10^{22}}{Z(Z+1) \cdot \ln(183 \cdot Z^{-1/3}) \cdot (-\ln\epsilon - \frac{5}{8}) \cdot N_g[m^{-3}]}$$

$$N_g[m^{-3}] = \frac{\rho[kg/m^3]}{2Z \cdot m_p[kg]} \cdot \frac{p[mbar]}{1013}$$

$$\tau_B[sec] = \frac{0.0008019}{(-\ln\epsilon - \frac{5}{8}) \cdot p[mbar]} \quad for\ Z = 7$$

$(Example:\ \tau_B = 29890sec = 8.30h;\ Z = 7(nitrogen);\ \epsilon = 0.0025;$
$N_g = 2.6347681 \cdot 10^{14} m^{-3};\ \rho = 1.25kg/m^3;\ p = 5 \cdot 10^{-9} mbar)$

Beam lifetime due to elastic electron scattering with residual gas nucleons:

$$\tau_{el} = \frac{2E^2}{\pi \cdot c \cdot E_0^2 \cdot (2r_e \cdot Z)^2 \cdot (\frac{<\beta_x>}{A_z} + \frac{<\beta_z>}{A_z}) \cdot N_g}$$

$$\tau_{el}[sec] = \frac{2.5603272 \cdot 10^{26} \cdot E^2}{Z^2 \cdot (\frac{<\beta_x>}{A_z} + \frac{<\beta_z>}{A_z}) \cdot N_g[m^{-3}]}$$

$$N_g[m^{-3}] = \frac{\rho[kg/m^3]}{2Z \cdot m_p[kg]} \cdot \frac{p[mbar]}{1013}$$

$$\tau_{el}[sec] = \frac{99.16 \cdot E^2}{(\frac{<\beta_x>}{A_x} + \frac{<\beta_z>}{A_z}) \cdot p[mbar]} \quad for\ Z = 7$$

$(Example:\ \tau_{el} = 112070sec = 31.13h;\ Z = 7(nitrogen);\ \beta_x = 20m/rad;$
$A_x = 80 \cdot 10^{-6} m;\ \beta_z = 20m/rad;\ A_z = 6 \cdot 10^{-6} m;$
$N_g = 2.6347681 \cdot 10^{14} m^{-3};\ \rho = 1.25kg/m^3;\ p = 5 \cdot 10^{9} mbar)$

Cavity power:

$$P = \frac{U^2}{2R_s}$$

$(Example:\ P = 110kW;\ U = 1.755MV;\ R_s = 14M\Omega)$

Dispersion in a cavity leads to a displacement of the closed orbit during acceleration or deacceleration and therefore to an excitation of radial betatron oscillations

Polarization time in a storage ring (Sokolov-Ternov):

$$P_{max} = \frac{8}{5\sqrt{3}} = 0.9237604$$

$$\tau = \frac{8}{5\sqrt{3}} \cdot \frac{E_0^6 \cdot R \cdot r^2}{\hbar^2 \cdot c^2 \cdot r_e \cdot E^5}$$

$$\tau[sec] = 98.65992 \cdot \frac{R[m] \cdot r[m]^2}{E[GeV]^5} \quad for\ electrons$$

$(Example: \ \tau = 365sec; \ R = 46.0276m; \ r = 12.1849m; \ E = 4.5GeV)$
$(Example: \ \tau = 1513sec; \ R = 1008m; \ r = 608m; \ E = 30GeV)$

Luminosity:

$$L = \frac{1}{4\pi \cdot e^2 \cdot f_u} \cdot \frac{I_1 \cdot I_2}{\sigma_x \cdot \sigma_z}$$

$(Example: \ L = 2.49 \cdot 10^{28} cm^{-2} sec^{-1}; \ I_1 = 1mA; \ I_2 = 1mA;$
$f_u = 1.037MHz; \ \sigma_x = 0.075cm; \ \sigma_z = 0.0016cm; \ (\kappa = 1\%))$

Vertical tune shift at one interaction point (IP) in a storage ring (tune splitting; only the higher tune moves):

$$\Delta Q_z = \frac{r_e \cdot E_0 \cdot N_{bunch} \cdot \beta_z(IP)}{2\pi \cdot E \cdot \epsilon_x \cdot (\sqrt{\kappa \cdot \beta_x(IP) \cdot \beta_z(IP)} + \kappa \cdot \beta_z(IP))}$$

$$\Delta Q_z \approx \frac{r_e \cdot E_0 \cdot N_{bunch}}{2\pi \cdot E \cdot \sqrt{\epsilon_x \cdot \epsilon_z}} \cdot \sqrt{\frac{\beta_z(IP)}{\beta_x(IP)}}$$

$(Example: \ \Delta Q_z = 0.038; \ N_{bunch} = 2.5 \cdot 10^{11}; \ \beta_z(IP) = .09m;$
$E = 17GeV; \ \epsilon_x = 2 \cdot 10^{-7}m; \ \kappa = .013; \ \beta_x(IP) = 1.3m)$
$(Example: \ \Delta Q_z = 0.022; \ N_{bunch} = 2.7 \cdot 10^{11}; \ \beta_z(IP) = .04m;$
$E = 5.3GeV; \ \epsilon_x = 0.56 \cdot 10^{-6}m; \ \kappa = .05; \ \beta_x(IP) = .63m)$
$(Example: \ \Delta Q_z = 0.040; \ N_{bunch} = 2.5 \cdot 10^{10}; \ \beta_z(IP) = .04m;$
$E = 2GeV; \ \epsilon_x = 8.9 \cdot 10^{-8}m; \ \kappa = .03; \ \beta_x(IP) = .8m)$

Synchrotron frequency:

$$f_s = f_u \cdot \sqrt{\frac{H \cdot \alpha_c \cdot e \cdot U_{HF} \cdot \cos\psi_s}{2\pi \cdot E}}$$

$(Example: \ f_s = 39.9kHz; \ f_u = 1.037MHz; \ H = 482; \ \alpha_c = 0.016;$
$U_{HF} = 6MV; \ \psi_s = 20°; \ E = 4.5GeV)$

Radiated power of a particle with charge q:

$$P = \frac{c \cdot q^2 \cdot E^4}{6\pi \cdot \epsilon_0 \cdot E_0^4 \cdot r^2}$$

$$P[W] = 6.762567 \cdot 10^{-7} \frac{E[GeV]^4}{r[m]^2} \quad for\ electrons$$

$(Example: \ P = 0.35mW; \ E = 50GeV; \ r = 110m)$

Radiated energy of an electron (per turn):

$$E_{ph} = \frac{e^2 \cdot E^4}{3\epsilon_0 \cdot E_0^4 \cdot r} = \frac{4\pi}{3} \cdot \frac{r_e}{E_0^3} \cdot \frac{E^4}{r}$$

$$E_{ph}[keV] = 88.46270 \cdot \frac{E[GeV]^4}{r[m]} \quad for\ electrons$$

$(Example: \ E_{ph} = 2.977MeV; \ E = 4.5GeV; \ r = 12.1849m)$
$(Example: \ E_{ph} = 5.729MeV; \ E = 5.3GeV; \ r = 12.1849m)$
$(Example: \ E_{ph} = 5.03GeV; \ E = 50GeV; \ r = 110m)$

Radiated synchrotron radiation power in a storage ring:

$$P = \frac{E_{ph}}{e} \cdot I; \quad I = \frac{N_e \cdot e \cdot c}{U}$$

$$P = \frac{e}{3\epsilon_0 \cdot E_0^4} \cdot \frac{E^4}{r} \cdot I = \frac{4\pi \cdot r_e}{3e \cdot E_0^3} \cdot \frac{E^4}{r} \cdot I$$

$$P[kW] = 88.46270 \cdot \frac{E[GeV]^4}{r[m]} \cdot I[A] \quad for\ electrons$$

$(Example: \ P = 297.7kW; \ E = 4.5GeV; \ r = 12.1849m; \ I = 0.1A)$
$(Example: \ P = 572.9kW; \ E = 5.3GeV; \ r = 12.1849m; \ I = 0.1A)$
$(Example: \ P = 574.0kW; \ E = 12GeV; \ r = 191.7295m; \ I = 0.06A)$

Radiated power of a bending magnet:

$$P = \frac{2 \cdot r_e}{3e \cdot E_0^3} \cdot L \cdot I \cdot \frac{E^4}{r^2} = \frac{2 \cdot c^2 \cdot r_e \cdot e}{3E_0^3} \cdot L \cdot I \cdot E^2 \cdot B^2$$

$$P[kW] = 14.07928 \cdot L[m] \cdot I[A] \cdot \frac{E[GeV]^4}{r[m]^2}$$

$$= 1.265382 \cdot L[m] \cdot I[A] \cdot E[GeV]^2 \cdot B[T]^2 \quad for\ electrons$$

$(Example: \ P = 23.87kW; \ L = 3.19m; \ I = 0.1A;$
$E = 5.3GeV; \ r = 12.1849m)$

Radiated power of a wiggler or undulator with sinusoidal field:

$$P[kW] = \frac{1.265382}{2} L[m] \cdot I[A] \cdot E[GeV]^2 \cdot B_{max}[T]^2 \quad for\ electrons$$

$(Example: \ P = 0.659kW; \ L = 2.112m; \ I = 0.1A;$
$E = 3.7GeV; \ B_{max} = 0.60T)$
$(Example: \ P = 5.228kW; \ L = 4.000m; \ I = 0.1A;$
$E = 4.5GeV; \ B_{max} = 1.01T)$
$(Example: \ P = 1.696kW; \ L = 2.4m; \ I = 0.045A;$
$E = 5.3GeV; \ B_{max} = 0.94T)$
$(Example: \ P = 6.614kW; \ L = 4.000m; \ I = 0.06A;$
$E = 12GeV; \ B_{max} = 0.55T)$

Central power density of a bending magnet for $\epsilon = 0$:

$$P = \frac{1}{0.608} \cdot \frac{2 \cdot r_e}{3 \cdot \sqrt{2\pi} \cdot e \cdot E_0^4} \cdot \frac{E^5}{r} \cdot I$$

$$P[W/mrad^2] = 18.1 \cdot \frac{E[GeV]^5}{r[m]} \cdot I[A] \;\; for\ electrons$$

$(Example:\ P = 0.274kW/mrad^2;\ E = 4.5GeV;\ r = 12.1849m;\ I = 0.1A)$

Central power density of a wiggler or undulator for $\epsilon = 0$:

$$P[W/mrad^2] = 10.85 \cdot N \cdot E[GeV]^4 \cdot B_{max}[T] \cdot I[A] \;\; for\ electrons$$

$(Example:\ P = 45.2kW/mrad^2;\ N = 127;\ E = 4.5GeV;$
$B_{max} = 0.8T;\ I = 0.1A)$
$(Example:\ P = 94.3kW/mrad^2;\ N = 127;\ E = 12GeV;$
$B_{max} = 0.55T;\ I = 0.06A)$

Critical wave length of the synchrotron radiation:

$$\lambda_c = \frac{4\pi}{3} \cdot \frac{E_0^3}{c \cdot B \cdot E^2} = \frac{4\pi}{3} \cdot \frac{r \cdot E_0^3}{E^3}$$

$$\lambda_c[\mathring{A}] = \frac{18.64353}{B[T] \cdot E[GeV]^2} = 5.589191 \cdot \frac{r[m]}{E[GeV]^3} \;\; for\ electrons$$

$(Example:\ \lambda_c = 0.747\mathring{A};\ r = 12.1849m;\ E = 4.5GeV)$
$(Example:\ \lambda_c = 0.457\mathring{A};\ r = 12.1849m;\ E = 5.3GeV)$
$(Example:\ \lambda_c = 0.1616\mathring{A};\ r = 608m;\ E = 27.6GeV)$

Critical energy of the synchrotron radiation:

$$E_c = \frac{3}{2} \cdot \frac{\hbar \cdot c}{E_0^3} \cdot \frac{E^3}{r} = \frac{2\pi \cdot \hbar \cdot c}{\lambda_c}$$

$$E_c[eV] = 2218.286 \cdot \frac{E[GeV]^3}{r[m]} = \frac{12398.42}{\lambda_c[\mathring{A}]} \;\; for\ electrons$$

$(Example:\ E_c = 16589eV;\ E = 4.5GeV;\ r = 12.1849m)$
$(Example:\ E_c = 27103eV;\ E = 5.3GeV;\ r = 12.1849m)$
$(Example:\ E_c = 19993eV;\ E = 12GeV;\ r = 191.7295m)$
$(Example:\ E_c = 76708eV;\ E = 27.6GeV;\ r = 608m)$

Fundamental wave length and energy of an undulator:

$$\lambda_{ph} = \frac{\lambda}{2\gamma^2} \cdot (1 + \frac{k^2}{2} + \gamma^2 \cdot \theta^2); \quad \gamma = \frac{E}{E_0}$$

$$\theta \;=\; observation\ angle\ with\ respect\ to\ the\ undulator\ beam$$

$$E_{ph}[eV] = \frac{12398.42}{\lambda_{ph}[\mathring{A}]}$$

$(Example: \ \lambda_{ph} = 4.648\mathring{A};\ \lambda = 3.14cm;\ E = 4.5GeV;\ k = 1.61;\ \theta = 0)$
$(Example: \ E_{ph} = 2667eV;\ \lambda_{ph} = 4.648\mathring{A})$
$(Example: \ \lambda_{ph} = 0.6537\mathring{A};\ \lambda = 3.14cm;\ E = 12GeV;\ k = 1.61;\ \theta = 0)$
$(Example: \ E_{ph} = 18967eV;\ \lambda_{ph} = 0.6537\mathring{A})$

Undulator beam divergence for the i^{th} harmonics:

$$\sigma'_{ph} = \sqrt{\frac{1 + \frac{k^2}{2}}{2i \cdot N \cdot \gamma^2}}$$

$(Example: \ \sigma'_{ph} = 10.8\mu rad;\ k = 1.61;\ i = 1;\ N = 128;\ \gamma = 8806)$
$(Example: \ \sigma'_{ph} = 2.33\mu rad;\ k = 1.61;\ i = 3;\ N = 128;\ \gamma = 23483)$

Undulator beam divergence (fundamental mode; zero electron emittance):

$$\sigma'_{ph} = \sqrt{\frac{\lambda_u \cdot (1 + \frac{k^2}{2})}{2\gamma^2 \cdot L}}$$

$$\sigma'_{ph} = \sqrt{\frac{\lambda_{ph}}{L}}; \quad \lambda_{ph} = \frac{\lambda_u}{2\gamma^2}(1 + \frac{k^2}{2})$$

$(Example: \ \sigma'_{ph} = 6.12\mu rad;\ \lambda_{ph} = 0.15nm;\ L = 4m)$

Undulator beam size for zero electron emittance:

$$\sigma_{ph} = \frac{1}{4\pi} \cdot \sqrt{\lambda_{ph} \cdot L}$$

$(Example: \ \sigma_{ph} = 31.5\mu m;\ \lambda_{ph} = 6.28nm;\ L = 25m)$

Photon beam emittance for zero electron emittance:

$$\epsilon_{ph} = \frac{\lambda_{ph}}{4\pi}$$

$(Example: \ \epsilon_{ph} = 0.5nmrad;\ \lambda_{ph} = 6.28nm)$

Photon beam coherence angle:

$$\sigma'_{coh} = \frac{\lambda_{ph}}{4\pi \cdot \sigma_{total}}; \quad \sigma_{total} = \sqrt{\sigma_e^2 + \sigma_{ph}^2}$$

$(Example:\ \sigma'_{coh} = 362nrad;\ \lambda_{ph} = 0.15nm;\ \sigma_{total} = 33\mu m)$

Undulator line width for the N^{th} harmonics and for n periods:

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{n \cdot N}$$

$(Example:\ \Delta\lambda/\lambda = 0.26\%;\ n = 128;\ N = 3)$

Undulator line broadening because of the electron beam divergence:

$$\frac{\Delta\lambda}{\lambda} = \frac{(\gamma \cdot \sigma'_r)^2}{1 + \frac{k^2}{2}}$$

$(Example:\ \Delta\lambda/\lambda = 8.8\%;\ E = 4.5GeV; \sigma'_r = 51\mu rad;\ k = 1.61)$

Undulator line broadening because of the acceptance angle θ:

$$\frac{\Delta\lambda}{\lambda} = \frac{(\gamma \cdot \theta)^2}{1 + \frac{k^2}{2}}$$

Undulator line broadening because of the electron energy spread:

$$\frac{\Delta\lambda}{\lambda} = 2\frac{\Delta E}{E}$$

$(Example:\ \Delta\lambda/\lambda = 0.22\%;\ \Delta E/E = 0.11\%)$

Divergence of synchrotron radiation from a bending magnet (vertical plane):

$$\theta = 0.608 \cdot \frac{E_0}{E} = \frac{0.608}{\gamma}$$

$$\theta = \frac{E_0}{E} \cdot (\frac{\lambda}{\lambda_c})^{1/3}\ for\ \lambda >> \lambda_c; \quad \lambda > 507 \cdot \lambda_c$$

$$\theta = 0.565 \cdot \frac{E_0}{E} \cdot (\frac{\lambda}{\lambda_c})^{0.425}\ for\ \lambda > 0.750 \cdot \lambda_c$$

$$\theta = \frac{E_0}{E} \cdot (\frac{\lambda}{3\lambda_c})^{1/2}\ for\ \lambda < 0.750 \cdot \lambda_c$$

$(Example:\ \theta = 0.069mrad;\ E = 4.5GeV)$
$(Example:\ \theta = 0.514mrad;\ E = 4.5GeV;\ \lambda = 100\mathring{A};\ \lambda_c = 0.747\mathring{A})$
$(Example:\ \theta = 2.209mrad;\ E = 4.5GeV;\ \lambda = 5500\mathring{A};\ \lambda_c = 0.747\mathring{A})$

Divergence of synchrotron radiation from a bending magnet (horizontal plane):

$$\theta = \frac{7}{12} \cdot \frac{0.608}{\gamma}$$

Spectral flux (bending magnet):

$$I[phot./(sec\,mrad\,0.1\%\,bandw.)] = 2.458{\cdot}10^{10}\cdot I_e[mA]\cdot E_e[GeV]\cdot\frac{E}{E_c}\cdot\int_{E/E_c}^{\infty}K_{5/3}(\eta)d\eta$$

$$\int_1^{\infty}K_{5/3}(\eta)d\eta = 0.6522$$

$$(Example:\ I = 7.214\cdot 10^{12}phot./(sec\ mrad\ 0.1\%\ bandw.);$$
$$I_e = 100mA;\ E_e = 4.5GeV;\ E = E_c)$$

Spectral flux (wiggler):

$$I[phot./(sec\,mrad\,0.1\%\,bandw.)] = 2.458{\cdot}10^{10}{\cdot}2N{\cdot}I_e[mA]{\cdot}E_e[GeV]{\cdot}\frac{E}{E_c}{\cdot}\int_{E/E_c}^{\infty}K_{5/3}(\eta)d\eta$$

$$(Example:\ I = 2.02\cdot 10^{14}phot./(sec\ mrad\ 0.1\%\ bandw.);$$
$$N = 28;\ I_e = 100mA;\ E_e = 4.5GeV;\ E = E_c)$$

Spectral central brightness for $\epsilon = 0$ (bending magnet):

$$I[phot./(sec\,mrad^2\ 0.1\%\,bandw.)] = 1.325{\cdot}10^{10}\cdot I_e[mA]\cdot E_e[GeV]^2\cdot(\frac{E}{E_c})^2\cdot K_{2/3}(\frac{E}{2E_c})$$

$$K_{2/3}(\frac{1}{2}) \approx 1.45$$

$$(Example:\ I \approx 3.9\cdot 10^{13}phot./(sec\ mrad^2\ 0.1\%\ bandw.);$$
$$I_e = 100mA;\ E_e = 4.5GeV;\ E = E_c)$$

Spectral central brightness for $\epsilon = 0$ (wiggler):

$$I[phot./(sec\,mrad^2\ 0.1\%\,bandw.)] = 1.325{\cdot}10^{10}{\cdot}2N{\cdot}I_e[mA]{\cdot}E_e[GeV]^2{\cdot}(\frac{E}{E_c})^2{\cdot}K_{2/3}(\frac{E}{2E_c})$$

$$(Example:\ I \approx 2.2\cdot 10^{15}phot./(sec\ mrad^2\ 0.1\%\ bandw.);$$
$$N = 28;\ I_e = 100mA;\ E_e = 4.5GeV;\ E = E_c)$$

SASE FEL ρ parameter (1 dimensional theory):

$$\rho = (\frac{k^2\cdot r_e\cdot n_b\cdot\lambda_u^2\cdot JJ^2(k)}{32\pi\cdot\gamma^3})^{1/3}$$

$$n_b = \frac{n}{\sqrt{2\pi}^3\cdot\sigma_r^2\cdot\sigma_s}$$

$$\frac{\epsilon_n}{\gamma} < \frac{\lambda_{phot}}{4\pi}$$

$$\sigma_e < \frac{\rho}{2}$$

$(Example: \ \rho = 0.00222; \ k = 1.266; \ n_b = 3.17 \cdot 10^{21} m^{-3};$
$\lambda_u = 2.73cm; \ JJ(k) = 0.877; \ E = 1GeV; \ n = 6.24 \cdot 10^9;$
$\sigma_r = 50\mu m; \ \sigma_s = 50\mu m; \ \epsilon_n = 1\pi mm \cdot mrad;$
$\lambda_{phot} = 6.24nm; \ \sigma_e = 0.001)$

Saturation length of SASE FEL (1 dimensional theory):

$$L_s = \lambda_u / \rho$$

$(Example: \ L_s = 12.3m; \ \lambda_u = 2.73cm; \ \rho = 0.00222)$

Photon peak power of SASE FEL (1 dimensional theory):

$$P_e = \frac{c \cdot n \cdot E_e}{\sqrt{2\pi} \cdot \sigma_s}$$

$(Example: \ P_e = 2.4TW; \ n = 6.24 \cdot 10^9; \ E_e = 1GeV; \ \sigma_s = 50\mu m)$

$$P_{phot} = \rho \cdot P_e$$

$(Example: \ P_{phot} = 5.3GW; \ \rho = 0.00222; \ P_e = 2.4TW)$

Speed of light:

$$c = 2.99792458 \cdot 10^8 m/sec$$

Rest energy / rest mass:

$$E_0(electrons) = 0.51099906 MeV; \quad m_e = 9.1093897 \cdot 10^{-31} kg$$

$$E_0(protons) = 938.27231 MeV; \quad m_p = 1.6726231 \cdot 10^{-27} kg$$

$$1[kg] = 1[VAsec^3/m^2]$$

Classical electron radius:

$$r_e = 2.81794092 \cdot 10^{-15} m; \quad r_e = \frac{e^2}{4\pi \cdot \epsilon_0 \cdot E_0}$$

Electron charge magnitude:

$$e = 1.60217733 \cdot 10^{-19} Asec; \quad 1[eV] = 1.60217733 \cdot 10^{-19} [Wsec]$$

Permeability of free space:

$$\mu_0 = 4\pi \cdot 10^{-7} Vsec/Am$$

Permittivity of free space:

$$\epsilon_0 = 8.854187817 \cdot 10^{-12} Asec/Vm$$

$$\epsilon_0 \cdot \mu_0 \cdot c^2 = 1$$

Planck constant:

$$\hbar = 6.5821220 \cdot 10^{-22} MeVsec = 1.05457266 \cdot 10^{-34} Wsec$$

Fine structure constant:

$$\alpha = \frac{1}{137.0359895}; \quad \alpha = \frac{e^2}{4\pi \cdot \epsilon_0 \cdot \hbar \cdot c}$$

Gaussian curve:

$$f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}$$

$$f_{max}(x) = \frac{1}{\sqrt{2\pi}}$$

$$\int_{-0.5\cdot\sqrt{2\pi}}^{0.5\cdot\sqrt{2\pi}} f_{max} dx = 1$$

$$\sqrt{2\pi} = 2.506628$$

$$FWHM = 2\sqrt{2ln2} = 2.354820$$

$$\int_{-1}^{1} f(x)dx = 0.6827$$

$$\int_{-0.5\cdot FWHM}^{0.5\cdot FWHM} f(x)dx = 0.761$$

$$\int_{-2}^{2} f(x)dx = 0.9545$$

$$\int_{-3}^{3} f(x)dx = 0.9973$$